

# Anelastic properties of resonant transducers for cryogenic gravitational wave antennas

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## Abstract

Low anelastic losses are essential to obtain a good sensitivity of resonant cryogenic gravitational wave antennas. In the most common transduction scheme the energy left in the antenna by the gravitational wave is transferred to a small mass with the same resonant frequency. In this way it is possible to convert the mechanical excitation in an electrical signal with higher efficiency. The small resonator coupled to the large detection mass should have anelastic characteristics as good as those of the antenna in order not to degrade the sensitivity of the apparatus. Results of anelastic relaxation measurements performed on resonators of Al5056, grade 2 Ti and 38NiCrMo4 steel at a frequency of 1 kHz from room temperature down to 1.5 K are presented.

## 1. Introduction

A crucial element of a resonant gravitational wave (GW) detector is the electromechanical transducer which converts the mechanical vibrations of the massive elastic body (the GW antenna) into an electromagnetic signal. The sensitivity of the GW detector is strongly dependent on the performance of the transducer and on its coupling to the following low noise amplifier [1].

Many different and ingenious types of transducer have been proposed and a few of them have been tested in the laboratory coupled to a resonant antenna [2]. In this effort a simple method to improve the transducer sensitivity has been conceived. The basic idea is to obtain a mechanical amplification of the antenna vibration by constructing the sensitive element of the transducer with a mechanical frequency near or equal to the antenna resonance frequency. Then, operating with a frequency tuning, the vibration amplitude of the antenna is increased in the transducer by a factor equal to the square root of the ratio between the equivalent mass of the antenna and the equivalent mass of the transducer.

The general expression of the detector sensitivity is given in terms of the effective temperature [3]

$$T_{\text{eff}} = 2T_n \left[ \left( 1 + \frac{1}{\lambda^2} \right) \left( 1 + \frac{2\lambda T}{\beta Q T_n} \right) \right]^{1/2} \quad (1)$$

where  $T_n$  is the noise temperature of the amplifier,  $T$  is the thermodynamical temperature of the apparatus,  $Q$  is the overall quality factor of the vibrating system,  $\beta$  is an efficiency parameter defined as the ratio of the electromagnetic energy in the transducer to the total mechanical energy in the system, while the adimensional quantity  $\lambda$  is defined as the ratio between the noise impedance of the amplifier and the output impedance of the transducer. From this formula it follows that the minimum value of  $T_{\text{eff}}$ ,  $2T_n$ , is achieved when  $1/\lambda^2 \ll 1$  and  $2\lambda T/\beta Q T_n \ll 1$ , which formally represents the electromechanical matching in the system (antenna–transducer–amplifier).

While a noise temperature of the amplifier,  $T_n$ , in the range of  $10^{-6}$  K is provided by a superconducting quantum interference device (SQUID), a detector sensitivity of the same order, given by formula (1), can only be obtained by fulfilling the above two matching conditions. This implies the use of a detector cooled at very low temperatures ( $T < 100$  mK), with high values of both the quality merit factor ( $Q \approx 10^7$ ) and the energy-coupling coefficient ( $\beta \approx 10^{-2}$ ). However, the construc-

tion of a transducer with a well-maximized  $\beta Q$  value is a very difficult task. Provided that the transducer is mechanically resonating, it can be shown that the energy-coupling factor  $\beta$  depends on both the geometry of its vibrational mode and the elastic properties of the material for a given ratio of the equivalent mass of the transducer to the equivalent mass of the antenna ( $m_t/m_a$ ).

On the other hand, some of the parameters which are involved in the  $\beta$  optimization also play an important role in obtaining a high quality merit factor. It is even more difficult to obtain high  $Q$  values, because there are several dissipation mechanisms in the whole system, basically in the transducer, and some of them cannot be formally modelled. The elastic losses depend on the type of the transducer vibration mode, on the intrinsic dissipation of the material and on the mechanical assembling of its complete structure. Moreover, the residual stresses accumulated in the specimen during the machining procedure can also limit the overall  $Q$  value. Hence extensive experimental tests must be performed in order to characterize the anelastic behaviour of the transducer and the overall effect on mechanical losses of its non-resonating parts. However, to carry out this optimization, we had to develop a suitable test facility in order to characterize at low temperature the performances of the transducer [4]. Detailed studies on the elastic material properties are usually carried out by looking at a vibration mode of a sample with at least one nodal point. This node is used to suspend the sample, obtaining in this simple way a nearly perfect mechanical insulation. In the case of a transducer vibration mode the lack of nodal points does not permit one to realize such a configuration and the suspension system becomes a crucial part of the experimental set-up. The significant perturbation of the suspension on the  $Q$  and resonance frequency measurements must be evaluated and thus an accurate characterization of the suspension set-up is needed.

In the following section we describe this system, in Section 3 we describe the general layout of our transducer and in Section 4 we present some experimental data on mechanical  $Q$  measurements obtained during several runs carried out to optimize the GW transducer.

## 2. The suspension system

After several tests with various shapes the system in Fig. 1 has been developed. The test system is basically composed of a vacuum chamber (Fig. 1, upper view) inside which the transducer and its suspension system are located (Fig. 1, lower view).

Since the vacuum chamber is the element which finally connects the transducer with the bath of cryogenic

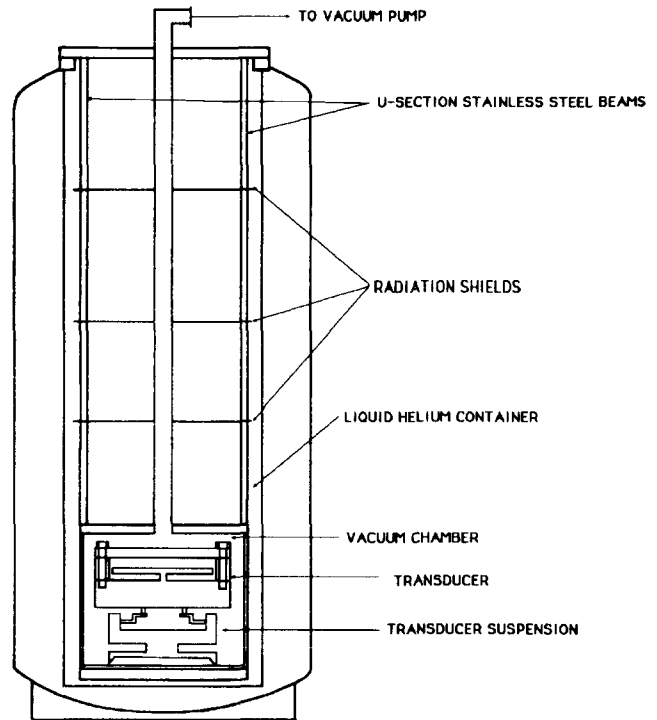


Fig. 1. General scheme of the test system.

liquids and with the cryostat, it must prevent as much as possible the residual damping of the mechanical energy transmitted by the suspension.

We found that a heavy and very rigid structure, well clamped to the outer shell of the cryostat, is preferable to a flexible one with a resonance frequency lower than that of the transducer. For this reason the vacuum chamber in our system is connected by means of four U-shaped stainless steel beams to the upper flange of the cryostat.

The suspension on which the transducer lies has the double-dead-bug geometry developed for the insulation from external acoustic and seismic noise of resonant gravitational wave antennas [5]. Other tests have been carried out with different suspension systems involving mechanical filters with heavier intermediate masses and different elements of suspension. In a few cases the performances we obtained were comparable with those of the double-dead-bug suspension: we decided on this set-up mainly because of its ease of use with respect to other filter configurations. A detailed analysis of the effect of the suspension set-up on the frequency and  $Q$  of the transducer under test is given in ref. 4.

## 3. The resonator

The general layout of our transducer for resonant cryogenic gravitational wave antennas is shown in

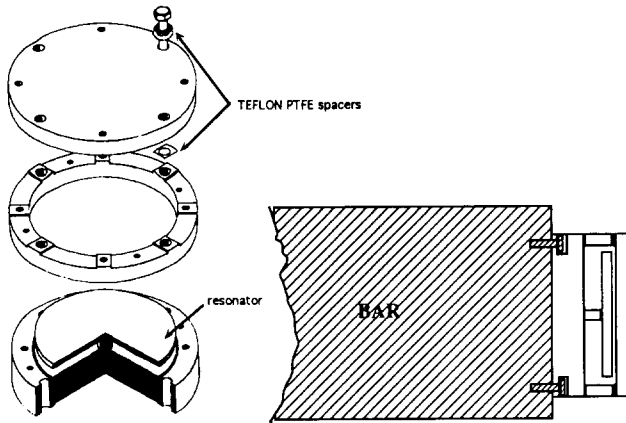


Fig. 2. Exploded view of the Al5056 transducer TRC/G6 and scheme of the transducer coupled to the bar.

Fig. 2. The resonator has the shape of a disk which is free at the outer edge and fixed at the inner edge. In order to have the maximum coupling with the mass of the antenna, the frequency of the first symmetrical flexural mode is made equal to the detection frequency (1 kHz), within a few hertz, by precision machining. The typical mass of the resonator is 300 g. The disk is connected by a shank to a heavy mass which contains the bolts to clamp the transducer to the end face of the bar.

To pick up the motion of the disk, a capacitor is formed by facing a plate to the surface of the resonator and a biasing voltage as high as possible is applied. Typical values of gap and applied electrical field are  $50 \mu\text{m}$  and  $5 \times 10^6 \text{ V m}^{-1}$  respectively. The transducer works at low temperature (from 4.2 to 0.1 K) and is usually charged at the beginning of operation and then insulated. The typical characteristic discharge time is about 1 year.

#### 4. $Q$ measurements of resonators: experimental results

##### 4.1. Measurements on Al5056 transducers

Usually the resonator is built using the same material as the bar, Al5056, an Al alloy (5% Mg) with very good anelastic properties at low temperature. In Fig. 3 we report the anelastic behaviour of two transducers built of Al5056 and tested in our laboratory. A maximum value of  $Q \approx 3 \times 10^6$  has been obtained at 4.2 K with the transducer coded PPT2, which had a non-standard electrode configuration [6]. We stress that the best elastic properties have been shown by PPT2 in spite of its more complex mechanical structure. The resonator of this transducer is a disk 17.0 cm in diameter and 0.65 cm thick which is fixed at the inner radius and free at the outer radius, vibrating in the first flexural

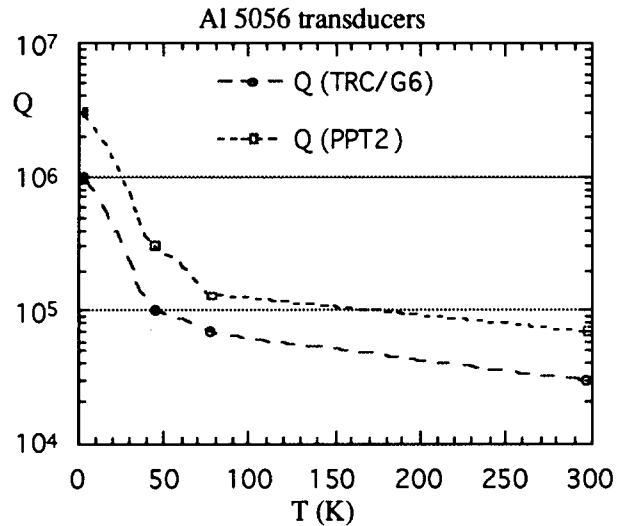


Fig. 3. Cryogenic test  $Q$  vs.  $T$  for two Al5056 transducers.

mode. The shank of the vibrating plate had a large diameter in its terminal part, which was threaded so as to be able to fix it to a heavy mass. The whole structure is made of Al5056. We point out that for the bulk material  $Q$  values up to  $4 \times 10^7$  have been measured at low temperature [7], an order of magnitude higher than those observed on our resonators, both in the laboratory and coupled to an antenna [8].

The reason for this discrepancy is not clear, more so because of the intrinsic difficulties in discriminating the anelastic effects from the dissipation effects due to clamping and suspension. Since the first appearance of this kind of resonator [9], the thermoelastic effect has been indicated as the possible dominant dissipative mechanism. Following this hypothesis, we have tested the anelastic properties of two transducers of the same standard configuration, built using low conductivity materials: the first transducer is made of 38NiCrMo4 steel and the second is made of grade 2 Ti.

##### 4.2. Measurements of 38NiCrMo4 steel transducer (TRC/R1)

38NiCrMo4 steel is a material widely used to construct elastic components in the aeronautic and locomotive industries.

The vibrating plate had a diameter of 170 mm and a thickness of 6.7 mm. Because of the high density of the material ( $\rho = 7.8 \text{ g cm}^{-3}$ ), we cannot take advantage of the high Young modulus value (206 GPa) to obtain a higher  $\beta$  value in comparison with the Al transducer. On the other hand, the suspension loaded by a higher sensor mass (22 kg) had a resonance frequency of the first stage so low that, in a first approximation, the suspension perturbation on the frequency measurement can be neglected. In Fig. 4 the quality factor of the steel sample vs. temperature is shown. We notice that the  $Q$  increase observed by lowering the temperature

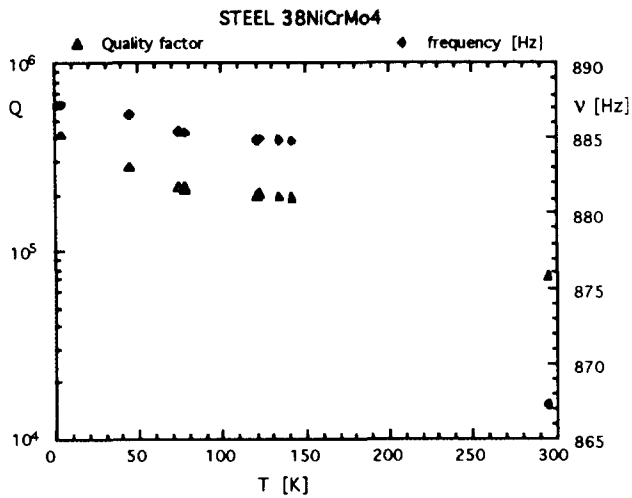


Fig. 4. Quality factor and resonance frequency drift vs. temperature for the 38NiCrMo4 steel transducer.

is less steep than for the Al alloy and that the maximum value attained at 1.5 K,  $Q = 6.5 \times 10^5$ , is not sufficient to provide an improvement in transducer performance.

#### 4.3. Measurements on a grade 2 Ti transducer (TRC/R3)

Grade 2 Ti (99.6% Ti) is a material which has lower density than the steel ( $\rho = 4.54 \text{ g cm}^{-3}$ ) and a Young modulus of 116 GPa; the combination of these two parameters is promising to obtain a suitable  $\beta Q$  product. In this sample the diameter and thickness of the vibrating plate were 170 mm and 6.2 mm respectively, the shank diameter was 15 mm and the first flexural symmetric resonance  $\nu_1^c$  was measured at 947.0 Hz at room temperature.

After cooling the sample to 4.2 K, we monitored the temperature, the resonance frequency and the quality factor during slow warming of the system. The monitoring was performed using the GPIB control interface of an HP 35660A dynamic signal analyser and the Lake-Shore DRC80 unit of a cryogenic thermometer. In Fig. 5 the measured resonance frequency of the first flexural mode of the mushroom coupled to the suspension and the  $Q$  factor vs. the temperature are reported; each datum point is obtained by averaging the results of 10 independent measurements. The maximum  $Q$  value,  $2.8 \times 10^6$ , occurs at 5 K in agreement with the results reported in the literature by Matacz *et al.* [10] who measured the acoustic losses of a sample held with a nodal point suspension at the frequencies of the asymmetrical flexural modes.

We point out that this result is a further test of the good performance of our suspension system as far as acoustic losses are concerned.

The acoustic absorption peak that appears in Fig. 5 is probably due to machining. Such peaks were first

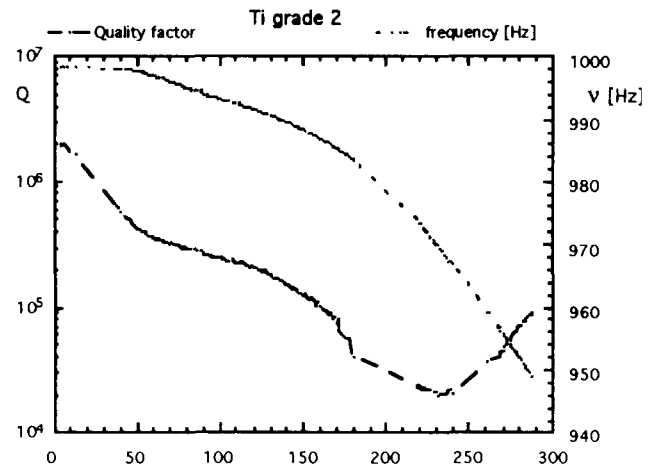


Fig. 5. Quality factor and resonance frequency drift vs. temperature for the grade 2 Ti transducer.

observed by Bordoni [11] in metals with f.c.c. lattice symmetry and depend mostly on the crystal dislocations created by a given amount of “cold work”. The peak peculiarity is its reproducibility in any thermal cycle under recrystallization.

We notice that at room temperature for both 38NiCrMo4 steel and grade 2 Ti the quality merit factor ( $Q \approx 10^4$ ) is considerably higher than that of the Al5056 samples, supporting the hypothesis that thermoelastic losses are relevant at room temperature and negligible at liquid helium temperature.

## 5. Conclusions

We have developed a test facility for resonant transducers to be coupled to cryogenic gravitational wave resonant antennas. The test system permits one to measure the frequency of the uncoupled transducer with a precision of the order of 1 Hz at 1 kHz. This is sufficient for tuning it correctly at the operating frequency of the antenna. Although the realization of the transducer suspension presents difficulties because of the lack of accessible nodal points, it is shown that the test system permits one to measure high  $Q$  values and to evaluate accordingly the transducer performance. The results of tests performed on transducers with standard geometry and low thermal conductivity materials (38NiCrMo4 steel and grade 2 Ti) are shown as well as the  $Q$  measurements obtained with an Al5056 transducer with a peculiar push-pull geometry. They seem to indicate that thermoelastic dissipation effects do not limit the performance of the transducer at low temperature.

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